10th Class 2017

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define radical equation.

Ans An equation involving expression under the radical sign is called a radical equation.

(ii) Write the equation in standard form:

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$

Ans Multiplying both sides by $x(x + 1)$,

$$x(x+1)\frac{x}{x+1} + x(x+1)\frac{x+1}{x} = 6x(x+1)$$

$$x(x) + (x+1)(x+1) = 6x(x) + 6x(1)$$

$$x^2 + x^2 + 1 + 2x = 6x^2 + 6x$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$0 = 6x^2 + 6x - 2x^2 - 2x - 1$$

$$0 = 4x^2 + 4x - 1$$

$$\Rightarrow \quad 4x^2 + 4x - 1 = 0$$

(iii) Define simultaneous equations.

Ans A system of equations having a common solution is called a system of simultaneous equations.

(iv) Evaluate: $(9 + 4\omega + 4\omega^2)^3$

Ans Given, $(9 + 4\omega + 4\omega^2)^3$

$$= (9 + 4(\omega + \omega^2))^3$$

$$= (9 + 4(-1))^3$$

$$= (9 - 4)^3$$

$$= 5^3$$

$$= 125$$

(v) Without solving, find the sum and the product of the roots of quadratic equation:

$$(l+m)x^2 + (m+n)x + n - l = 0$$

Ans Here: $a = l + m,\ b = m + n,\ c = n - l$

Let α, β be the roots of equation:

Sum of the roots:

$$S = \alpha + \beta = \frac{-b}{a} = -\frac{m+n}{l+m}$$

Product of the roots:

$$P = \alpha\beta = \frac{c}{a} = \frac{n-l}{l+m}$$

(vi) Use synthetic division to find the quotient and the remainder, when $(x^2 + 7x - 1) \div (x + 1)$.

Ans Let, $P(x) = x^2 + 7x - 1$

Here, $x - a = x + 1$

$x - x - 1 = a$

$-1 = a$

$\Rightarrow \boxed{a = -1}$

By synthetic division,

-1	1	7	-1
		-1	-6
	1	6	-7

The depressed equation is

$$x + 6 = 0$$

$\therefore$ Quotient $= Q(x) = x + 6$

and Remainder $= -7$

(vii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

(viii) Find fourth proportional: $4x^4, 2x^3, 18x^5$

Ans Let y be the fourth proportional, then

$$4x^4 : 2x^3 :: 18x^5 : y$$

Product of extremes = Product of means

$$(4x^4)\,y = (2x^3)(18x^5)$$

$$y = \frac{(2x^3)(18x^5)}{4x^4}$$

$$\boxed{y = 9x^4}$$

(ix) If $3(4x - 5y) = 2x - 7y$, find the ratio $x : y$.

Ans

$$3(4x - 5y) = 2x - 7y$$

$$12x - 15y = 2x - 7y$$

$$12x - 2x = -7y + 15y$$

$$10x = 8y$$

$$\frac{x}{y} = \frac{8}{10}$$

$$\frac{x}{y} = \frac{4}{5}$$

$$x : y = 4 : 5$$

3. Write short answers to any SIX (6) questions: (12)

(i) Define a rational fraction.

Ans An expression of the form $\frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomial in x with real coefficients and $D(x) \neq 0$, is called a rational fraction.

(ii) How can we make partial fractions of $\frac{7x-9}{(x+1)(x-3)}$.

Ans Let

$$\frac{7x-9}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)} \qquad \text{(i)}$$

Multiplying by $(x + 1)(x - 3)$, we get

$$7x - 9 = A(x - 3) + B(x + 1)$$

$$7x - 9 = Ax - 3A + Bx + B$$

$$7x - 9 = Ax + Bx - 3A + B$$
$$7x - 9 = (A + B)x - 3A + B$$

By comparing coefficients of x and constant terms,

$$A + B = 7 \quad \text{(ii)}$$
$$-3A + B = -9 \quad \text{(iii)}$$

By subtracting (iii) from (ii), gives

$$\begin{array}{r} A + B = 7 \\ \underset{+}{-} 3A \underset{-}{+} B = \underset{+}{-}9 \\ \hline 4A \qquad = 16 \end{array}$$

$$A = \frac{16}{4}$$
$$A = 4$$

Put $A = 4$ in (ii)

$$4 + B = 7$$
$$B = 7 - 4$$
$$B = 3$$

By putting values of A, B in (i), we get

$$\frac{7x - 9}{(x + 1)(x - 3)} = \frac{4}{(x + 1)} + \frac{3}{(x + 3)}$$

(iii) Define complement of a set.

Ans If U is a universal set and A is subset of U, then the complement of A is the set of those elements of U which are not contained in A and is denoted by A^c or A'.

(iv) Find a and b if (a – 4, b – 2) = (2, 1).

Ans

$a - 4 = 2$; $b - 2 = 1$

$a = 2 + 4$; $b = 1 + 2$

$\boxed{a = 6}$; $\boxed{b = 3}$

(v) Define domain and range of a relation.

Ans **Domain:**

Domain of relation denoted by DomR is the set consisting of all the first elements of each ordered pair in the relation.

Range:

Range of relation denoted by range R is the set consisting of all the second elements of each ordered pair in the relation.

(vi) Find $A \cap B$, if $A = \{2, 3, 5, 7\}$ and $B = \{3, 5, 8\}$.

Ans $A \cap B = \{2, 3, 5, 7\} \cap \{3, 5, 8\}$

$= \{3, 5\}$

(vii) The marks of seven students in Mathematics are as follows. Find Arithmetic Mean: 45, 60, 74, 58, 65, 63, 49.

Ans Let, X = Marks of students

$X = 45, 60, 74, 58, 65, 63, 49$

The Arithmetic Mean: $(\bar{X})$

$$\bar{X} = \frac{\Sigma X}{n}$$

$$= \frac{45 + 60 + 74 + 58 + 65 + 63 + 49}{7}$$

$$= \frac{414}{7}$$

$$\boxed{\bar{X} = 59.14}$$

(viii) Find geometric mean of 2, 4 and 8.

Ans

$$G.M = (2 \times 4 \times 8)^{1/3}$$

$$= (64)^{1/3}$$

$$= (4^3)^{1/3}$$

$$= 4$$

(ix) Define mode.

Ans Mode is defined as the most frequent occurring observation in the data.

4. Write short answers to any SIX (6) questions: (12)

(i) Define radian.

Ans The angle subtended at centre of the circle by an arc, whose length is equal to the radius of the circle is called one radian.

(ii) Express 225° into radian.

Ans

$$225^\circ = 225 \times \frac{\pi}{180}$$

$$= \frac{5\pi}{4} \text{ radians}$$

(iii) In a circle of radius 12 m, find the length of an arc which subtends a central angle $\theta = 1.5$ radian.

Ans

$r = 12$ m

$\theta = 1.5$ radian

$l = ?$

$l = r\theta$

$= (12)(1.5)$

$l = 18$ m

(iv) Define projection of a point.

Ans The projection of a given point on a line segment is the foot of perpendicular drawn from the point on that line segment.

(v) Define radical segment.

Ans Radical segment of a circle is a line segment, determined by the centre and a point on a circle.

(vi) Define the tangent to a circle.

Ans A tangent to a circle is the straight line which touches the circumference at a single point only.

(vii) **Define sector of a circle.**

Ans The circular region bounded by an arc of a circle and its two corresponding radical segments is called a sector of a circle.

(viii) **Define central angle.**

Ans The angle subtended by an arc at the centre of a circle is called its central angle.

(ix) **Define geometry.**

Ans Geometry is an important branch of mathematics, which deals with the shape, size and position of geometric figures.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: $2x + 5 = \sqrt{7x + 16}$ (4)

Ans Given equation:

$$2x + 5 = \sqrt{7x + 16}$$

Taking square on both sides,

$$(2x + 5)^2 = (\sqrt{7x + 16})^2$$

$$(2x)^2 + (5)^2 + 2(2x)(5) = 7x + 16$$

$$4x^2 + 25 + 20x = 7x + 16$$

$$4x^2 + 20x + 25 - 7x - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$4x^2 + 4x + 9x + 9 = 0$$

$$4x(x + 1) + 9(x + 1) = 0$$

$$(x + 1)(4x + 9) = 0$$

$$x + 1 = 0 \quad ; \quad 4x + 9 = 0$$

$$\boxed{x = -1} \quad ; \quad 4x = -9$$

$$\boxed{x = \frac{-9}{4}}$$

(b) Use synthetic division to find the values of l and m, if $(x + 3)$ and $(x - 2)$ are the factors of the polynomial $x^3 + 4x^2 + 2lx + m$. (4)

Ans Here $P(x) = x^3 + 4x^2 + 2lx + m$

and $x - a = x + 3$

$\Rightarrow$ $\boxed{a = -3}$

	1	4	$2l$	m
-3		-3	-3	$-6l + 9$
	1	1	$2l - 3$	$-6l + m + 9$

Since -3 is zero of polynomial, so remainder equal to zero.

$$-6l + m + 9 = 0 \qquad (1)$$

Again,

$$x - a = x - 2$$
$$-a = -2$$
$$a = 2$$

Again using synthetic division,

	1	4	$2l$	m
2		2	12	$4l + 24$
	1	6	$2l + 12$	$4l + m + 24$

Since 2 is zero of the polynomial, so remainder equal to zero.

$$4l + m + 24 = 0 \qquad (2)$$

By solving equation (1) and equation (2),

$$\begin{array}{r} -6l + m + 9 = 0 \\ \underset{-}{+}4l \underset{-}{+} m \underset{-}{+} 24 = 0 \\ \hline -\quad 10l - 15 = 0 \end{array}$$

$$-10l = 15$$

$$l = \frac{15}{-10}$$

$$l = \frac{-3}{2}$$

Put $l = \frac{-3}{2}$ in equation (1), we get

$$-6\left(\frac{-3}{2}\right) + m + 9 = 0$$

$$9 + m + 9 = 0$$

$$m + 18 = 0$$

$$\boxed{m = -18}$$

Thus $\boxed{l = \frac{-3}{2}}$, $\boxed{m = -18}$

Q.6.(a) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (a, b, c, d, e, f, $\neq 0$), then show that $\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$. **(4)**

Ans Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$

$$\frac{a}{b} = K, \quad \frac{c}{d} = K, \quad \frac{e}{f} = K$$

$$a = bK, \quad c = dK, \quad e = fK$$

$$\text{L.H.S} = \frac{a}{b}$$

$$= \frac{bK}{b}$$

$$= K \qquad (1)$$

$$\text{R.H.S} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{(bK)^2 + (dK)^2 + (fK)^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{b^2K^2 + d^2K^2 + f^2K^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{K^2(b^2 + d^2 + f^2)}{b^2 + d^2 + f^2}}$$

$$= \sqrt{K^2} = K \qquad (2)$$

From (1) and (2),

L.H.S = R.H.S

i.e., $\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$ Proved

(b) Resolve into partial fractions: $\frac{x-11}{(x-4)(x+3)}$. (4)

Ans $\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$

$x - 11 = A(x + 3) + B(x - 4)$ (i)

Put $x = 4$, $x = -3$ in (i)

Firstly,

$4 - 11 = A(4 + 3) + B(4 - 4)$

$-7 = A(7) + 0$

$\Rightarrow$ $7A = -7$

$\boxed{A = -1}$

And

$-3 - 11 = A(-3 + 3) + B(-3 - 4)$

$-14 = 0 + B(-7)$

$\Rightarrow$ $-7B = -14$

$\boxed{B = 2}$

So,

$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$

Q.7.(a) If U = {1, 2, 3, 4, ---, 10}, A = {1, 3, 5, 7, 9}, B = {1, 4, 7, 10} then prove that $B - A = B \cap A'$. (4)

Ans To show $B - A = B \cap A'$

L.H.S $= B - A$

$= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$

$= \{4, 10\}$ (1)

Now,

$A' = U - A$

$= \{1, 2, 3, 4, ----, 10\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

and

R.H.S $= B \cap A'$

$= \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$

$= \{4, 10\}$ (2)

From equation (1) and equation (2),

L.H.S = R.H.S

$B - A = B \cap A'$

(b) The marks of six students in mathematics are as follows. Determine variance: (4)

Students	1	2	3	4	5	6
Marks	60	70	30	90	80	42

Ans

X	X^2
60	3600
70	4900
30	900
90	8100
80	6400
42	1764
372	25664

$$\text{Variance} = S^2 = \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2$$

$$= \frac{25664}{6} - \left(\frac{372}{6}\right)^2$$

$$= 4277.33 - (62)^2$$

$$= 4277.33 - 3844$$

$$\boxed{S^2 = 433.33}$$

Q.8.(a) Prove that: $\frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} = 4\tan\theta\sec\theta$. (4)

Ans $\text{L.H.S} = \frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta}$

$$= \frac{(1 + \sin\theta)^2 - (1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{(1+\sin^2\theta+2\sin\theta)-(1+\sin^2\theta-2\sin\theta)}{(1)^2-(\sin\theta)^2}$$

$$= \frac{1+\sin^2\theta+2\sin\theta-1-\sin^2\theta+2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta}$$

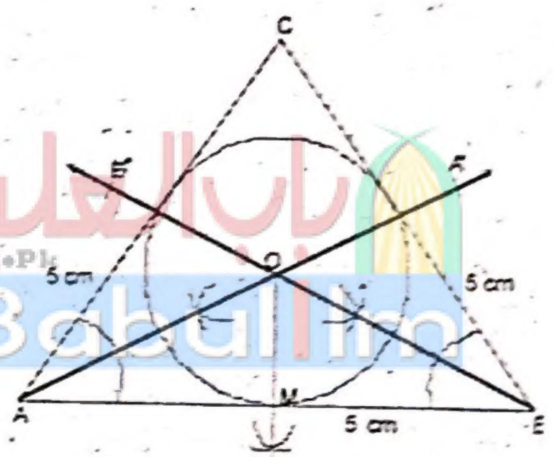
$$= 4\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$$

$$= 4\tan\theta\sec\theta$$

$$= \text{R.H.S}$$

(b) Inscribe a circle in an equilateral triangle ABC with each side of length 5 cm. (4)

Ans



Steps of Construction:

(i) Draw a ΔABC with each side = 5 cm.

(ii) Draw $\overrightarrow{AA'}$ bisector of $\angle A$.

(iii) Draw BE bisecter of $\angle B$.

$\overrightarrow{AA'}$ and $\overrightarrow{BE}$ intersect at point O.

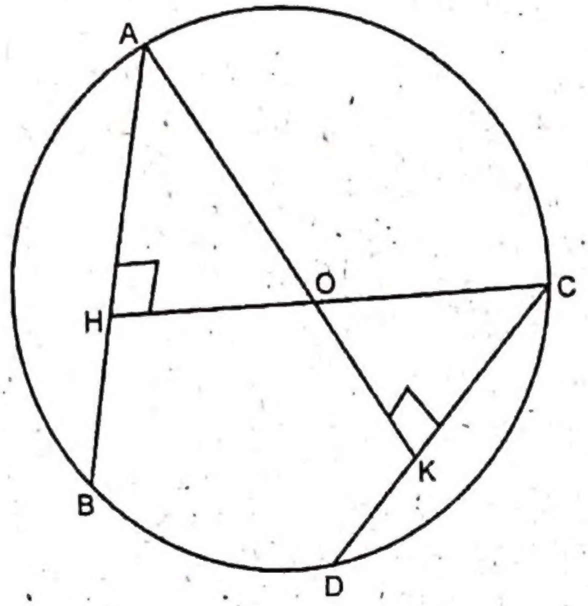
(iv) Drop $\overline{OM} \perp \overline{AB}$.

(v) Take O as centre and draw a circle with $m\overline{OM}$ as radius.

This is inscribed circle to triangle ABC.

Q.9. **Prove that two chords of a circle which are equidistant from the centre, are congruent. (4)**

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with center O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$.

To prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O, so that we can form $\angle$rt Δ^s OAH and OCK.

Proof:

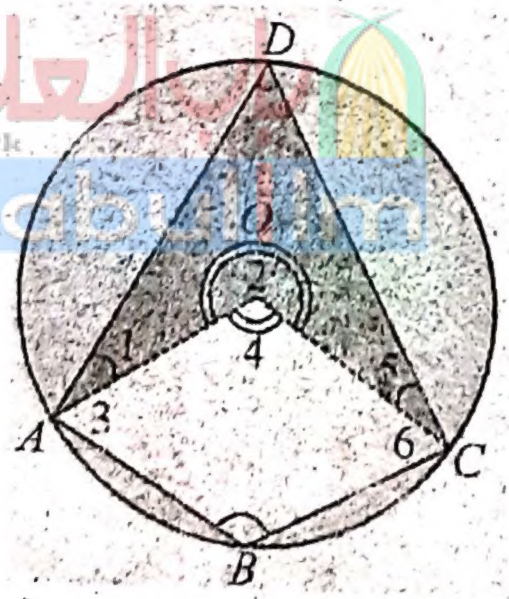
Statements	Reasons
In $\angle$rt Δ^s OAH $\leftrightarrow$ OCK	
$\because$ hyp. $\overline{OA}$ = hyp. $\overline{OC}$	Radii of the same circle
$m\overline{OH} = m\overline{OK}$	Given
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
So	
$m\overline{AH} = m\overline{CK}$ (i)	Corresponding sides of congruent triangles

But	
$m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly,	
$m\overline{CK} = \frac{1}{2} m\overline{CD}$ (iii)	$\overline{OK} \perp$ chord $\overline{CD}$ Given
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\therefore \frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (ii) and (iii)
or	
$m\overline{AB} = m\overline{CD}$	

OR

Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

Ans



Given:

ABCD is a quadrilateral inscribed in a circle with centre O.

To prove:

$$\begin{cases} m\angle A + m\angle C = 2\ \angle rts \\ m\angle B + m\angle D = 2\ \angle rts \end{cases}$$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$.
Write $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ as shown in the figure.

Statements	Reasons
Standing on the same arc ADC, $\angle 2$ is a central angle whereas $\angle B$ is the circumangle	Arc ADC of the circle with centre O.
$\therefore \quad m\angle B = \frac{1}{2}(m\angle 2) \quad$ (i)	By theorem 1
Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$ is the circumangle	Arc ABC of the circle with centre O.
$\therefore \quad m\angle D = \frac{1}{2}(m\angle 4) \quad$ (ii)	By theorem 1
$\Rightarrow \quad m\angle B + m\angle D = \frac{1}{2} m\angle 2 + \frac{1}{2} m\angle 4$ $= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$ (Total central angle) *i.e.*, $m\angle B + m\angle D = \frac{1}{2}(4\angle rt)$ $= 2\angle rt$ Similarly, $m\angle A + m\angle C = 2\angle rt$	Adding (i) and (ii)